The purpose of this page is to summarize frequently asked questions about the physics of skiing. The physics of skiing is a recurring topic on rsa. These calculations and interpretations were contributed to by a number of regulars, and some not so regulars, on rsa. The questions:

The following book on the physics of skiing is clear and fun to read. It's written at about the level of a freshman level physics course, and only occassionally uses much calculus, most of which is well motivated.

- David Lind and Scott P. Sanders, The physics of skiing : skiing at the triple point, Woodbury, N.Y., American Institute of Physics, 1997.

15.1 What's the radius of a pure carved turn?

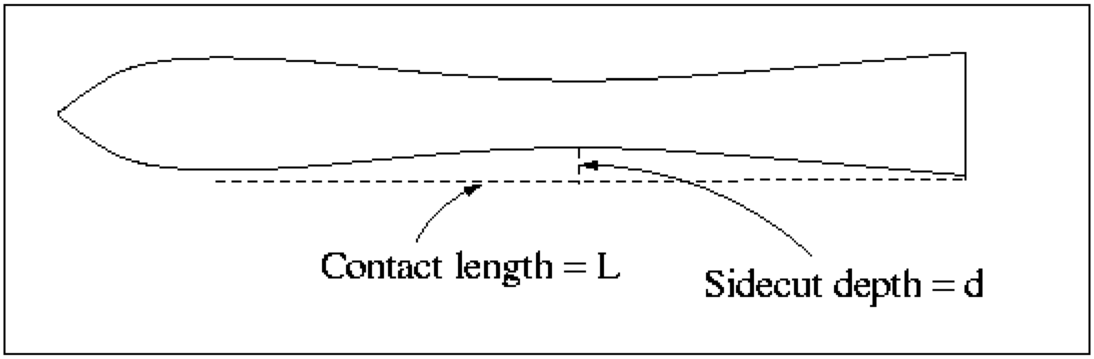
15.1.1 Sidecut radius of a ski

The sidecut radius of a ski is a design feature of a ski found by assuming the ski edge is an arc of a circle and determining the radius of that circle.

The following calculation descibes the radius of a circle if you know the parameters of a sector of the circle. A sector is found by cutting off the edge of a circle, or disc, with a straight cut that is perpendicular to the radius.

Assuming

- The ski edge forms the arc of the circle in the wedge.
- The contact line is the line running from the widest point on the tip of the ski to the widest point on the tail of the ski.
- The sidecut depth of the ski is the maximum height of the ski edge measured from the contact line.



define the variables

- L = length of the contact line
- d = sidecut depth
- Rsc = sidecut radius

then the sidecut radius is

- Rsc = (L^2/4 + d^2) / (2*d)

15.1.2 Turning radius of a ski

The radius of a turn is different, and it varies as the turn is being executed with the edge angle being used by the skier and with the amount of reverse camber in the ski during the turn. However, it is not too hard to calculate an approximation of the the radius of a "pure carved turn".

Geometrically, a carved turn is the composition of two curves, the sidecut of the ski and the deformed sidecut of the ski when its camber is reversed as it is pressed onto the snow. To find these curve, I assumed that the carving ski looks like a segment of a cylinder, and that the ski hill is an inclined plane. The arc the carving ski follows is the curve at the intersection of the cylinder and the inclined plane.

It will help you understand the formula if you'll do the following simple geometric demonstration. Take two identical rectangular pieces of paper. With scissors, make a large radius (shallow) sidecut on one of the long sides of piece 1. Make shorter radius (deeper) sidecut on the corresponding side of piece 2. Now hold each piece with the sidecut against a tabletop so that the edge of the sidecut is in full contact with the table. Notice the decambering effect of pushing the edge into contact with the table, and notice how changing the angle changes the radius of the approximate cylinder.

To use this approximation, you have to calculate the radius of the cylinder, and this involves the geometry of the ski. The approximation I make is not too bad if you assume that the ski does not twist, but it does calculate the radius of the carved turn when you are traversing the hill.

To approximate the cylinder radius, I computed the distance from the ski edge to a chord extended from the widest point on the ski tip to the widest point on the ski tail. Then I projected that point to the inclined plane given the inclination angle of the pressing force. If you think about this, its easy to see that there will be a minimum angle where this actually can be done. Thus all these calculations should be only considered for angles above that minimum.

The variables are:

- phi = angle swept out by the sidecut of the ski, i.e. Rsc*phi is the length of the edge from the widest point on the tip to the widest point on the tail.
- d_rc = reverse camber distance, distance from the flat ski to the edge when the ski is pushed into the snow
- alpha = inclination angle of the skiers leg.
- theta = inclination angle of the slope.
- Rcyl = cylinder radius.
- Rturn = turn radius

Here is the result

- d = Rsc * (1-cos(phi/2))
- d_rc = d * cot(alpha-theta)
- Rcyl = (L^2 + 4*d_rc^2)/(8*d_rc)
- Rturn = Rcyl * sin(alpha+theta)

15.1.3 Interpreting the formula

This formula is messy, but the interpretation of the formula is interesting and easily understood. It explains, mathematically, how a deep side cut ski carves easily. To understand this, I'll make a comparison between two different skis.

Suppose that you want to compare carved turns between a super sidecut ski and a regular ski when the angle of the skiers leg and the slope angle are the equal for both skis. (Keeping the angles the same makes for a fair comparison.)

For a ski with a large side cut to carve, it must be pressed a long way down to the snow before the edge is actually on the snow surface. In ski jargon, the ski has a lot of reversed camber when the edge is on the snow surface. This creates a small radius carved turn because the ski is bent into a small radius circular arc.

A ski with a small side cut cannot be pressed as far down before it comes in contact with the snow. Consequently, it will not have as much reversed camber. Therefore, the radius of the turn will be greater because the ski is bent into a larger radius circular arc.

A combination of the sidecut, edge angulation and reversed camber determine the radius of the turn. Skis with a large sidecut allow the skier to deeply reverse camber the ski, while skis with a small side cut cannot reverse camber as much. The combination of a large sidecut and a deep reverse camber creates short radius carved turns.

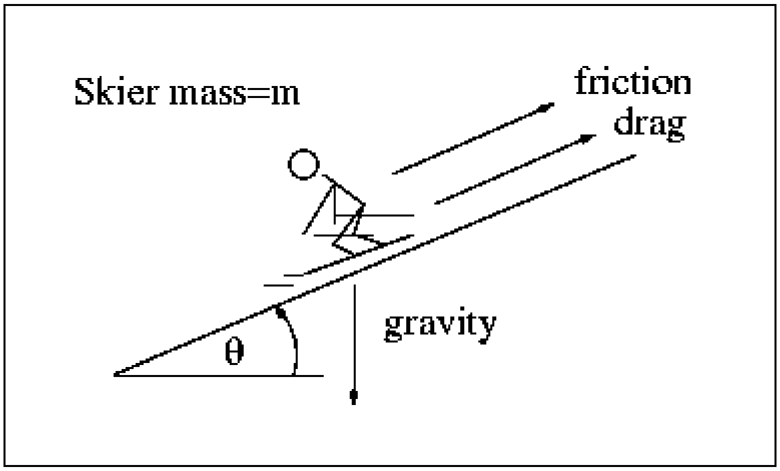
15.1.4 Is the a single turn radius of carved turn for a given ski?

No. The carve radius depends on the relative angle between the skiers leg and the hill. Of course, the design of some skis favor on turn radius over another, but there isn't a "single radius of a carved turn".

15.2 Do big people go faster?

All other things taken equal, the answer is yes, but it depend on how fast you are going.

From Patrick Chase (edited by Eyre): Let's look briefly at the free body diagram of a skier acted upon by gravity, surface friction with the snow (modeled as Coulombic friction) and aerodynamic body drag (neglecting skin friction). Here's the simple free body diagram.



Now we can look at the individual terms:

- m*A = inertial (body) forces
- Ffriction = mu*m*g*cos(theta)
- Fdrag = (Cd*Ap*rho*V^2)/2 - drag force, opposes gravity
- Fgravity = m*g*sin(theta) - gravitational force

The force Fgravity is the gravitational force. The inertial forces are zero if the skier is stopped or is moving at constant velocity. Ffriction and Fdrag are the frictional and drag forces. Both of these oppose gravity. The remaining terms are defined as

- A - acceleration
- Cd*Ap*rho - drag coefficient times frontal area times air density
- g - gravitational acceleration
- mu - dynamic friction coefficient
- V - velocity

Balancing these forces gives the equation of motion:

m*A = m*g*sin(theta) - mu*m*g*cos(theta) - (Cd*Ap*rho*V^2)/2

and if we divide by mass, we get

A = g*sin(theta) - mu*g*cos(theta) - (Cd*Ap*rho*V^2)/(2*m)

The only term depending on mass is the last term, and mass appears in the denominator. That term opposes gravity because (Cd*Ap*rho*V^2/m) is not negative.

It should be pretty clear that the benefits (or lack thereof) of changing mass depend upon what happens to (Cd*Ap/m) as mass is scaled upwards. It is expected that the frontal area will increase for a more massive skier, but it seems likely that Cd*Ap increases more slowly than m. Therefore, a bigger skier is likely to go faster.

A few important things in this analysis have been ignored:

- A heavier skier will probably have more trouble turning, especially if the added weight doesn't take the form of muscle in the right places.

- The friction at the ski/snow interface was treated as coulombic. The friction is generated in a film of meltwater in almost pure shear. So maybe the friction model is wrong, but this does not impact on the importance of mass.

15.2.1 If you're going fast?

The following data (from Stuart Marlatt in rec.climbing, edited by Eyre) measures the relative effects of changing (Cd*Ap/m). While these calculations were done for free falling objects, the effects would be similar for skiers as (Cd*Ap/m) is changed.

To determine the terminal velocity, I used a second order Runge-Kutta to integrate the equations of motion:

dz/dt = v
dV/dt = -g(z) + rho(z)*V^2*Cd(z,V)*Ap/2m

where

- z is the altitude
- V the velocity
- g is the acceleration of gravity as a function of altitude and latitude
- rho is the density of air (1962 Standand Atmosphere variation)
- Cd is the drag coefficient (as a function of altitude and velocity),
- Ap is the frontal area,
- m is the mass of our falling climber.

The fall is assumed to be from 1000m, at 41deg north. For reference a smooth sphere with a diameter about 0.5m was used in addition to three body positions where Cd*Ap were found from S. Hoerner's "Fluid Dynamic Drag":

- Cd*Ap = 0.11 for an upright body, minimal frontal area
- Cd*Ap = 0.84 for a horizontal body, maximal frontal area

- Cd*Ap = 0.46 for a body in tuck position

The latter two are the most applicable to skiers. Marlatt found the following results:

Position	Time of Fall	Speed at Impact	
Smooth sphere	15.76s	105.8 m/s	(236.7 mph)
Upright	16.44	95.3 m/s	(213.2 mph)
Horizontal	28.89	37.4 m/s	(83.7 mph)
Tuck	23.02	50.7 m/s	(113.4 mph)

15.2.2 If your going slow?

In reference to the description above, the question of how much faster does a big person go when you're going slow boils down to the following question:

How big is (Cd*Ap*rho*V^2/2m) relative to the other terms?'

Here is the answer. Suppose that you are free falling, i.e. no friction and theta = -90 degrees. Then you can calculate Cd from Patrick's third equation at terminal velocity (i.e. A = 0). This gives

Cd = 2*g*m/(Ap*rho*(V_T^2))

for the drag coefficient where V_T is the terminal velocity. By substituting this expression for Cd into Patrick's third equation you find

A = g*sin(theta) - mu*g*cos(theta) - g*[V/V_T]^2

The acceleration depends on how fast the person is going relative to his terminal velocity, i.e. on the ratio of V to V_t. The previous section showed v_T depends on the size and shape of the person. However, a slow skier is moving at only a small fraction of their terminal velocity, say maybe about 1/10.

In this case, (1/10)^2 = 0.01. The sine of a 20 degree slope is 0.342, so the acceleration due to the slope and gravity is 34 times greater than the effects due to mass! For most practical purposes, you can ignore the last term at small speeds.

15.3 A simplified mathematical model of avalanches